

Method of reduction to a steady wave; Uniqueness of motion by the energy integral method; Examples. 3. *Waves in membranes*: The governing differential equation; Solution for a rectangular membrane; Normal coordinates for a rectangular membrane; Circular membrane; Uniqueness of solutions; Examples. 4. *Longitudinal waves in bars and springs*: Differential equation for waves along a bar; Free vibrations of a finite bar; Vibrations of a clamped bar; Normal coordinates; Case of a bar in a state of tension; Vibrations of a loaded spring; Waves in an anharmonic lattice; Examples. 5. *Waves in liquids*: Summary of hydrodynamical formulae; Tidal waves and surface waves; Tidal waves, general conditions; Tidal waves in a straight channel; Tidal waves on lakes and tanks; Tidal waves on rectangular and circular tanks; Paths of particles; Method of reduction to a steady wave; Surface waves, the velocity potential; Surface waves on a long rectangular tank; Surface waves in two dimensions; Paths of the particles; The kinetic and potential energies; Rate of transmission of energy; Inclusion of surface tension. General formulae; Capillary waves in one dimension; Examples. 6. *Sound waves*: Relation between pressure and density; The governing differential equation; Solutions for a pipe of finite length; Normal modes; Normal modes in a tube with moveable boundary; The velocity potential. General formulae; The differential equation of wave motion; An example; Spherical symmetry; The kinetic and potential energies; Progressive waves in a tube of varying section; Examples. 7. *Electric waves*: Maxwell's equations; Non-conducting media and the wave equation; Electric and magnetic potentials; Plane polarised waves in a dielectric medium; Rate of transmission of energy in plane waves; Reflection and refraction of light waves; Internal reflection; Partially conducting media, plane waves; Reflection from a metal; Radiation pressure; Skin effect; Propagation in waveguides; Examples. 8. *General considerations*: Doppler effect; Beats; Amplitude modulation; Group velocity; Motion of wave packets; Kirchhoff's solution of the wave equation; Fresnel's principle; Diffraction at a pin hole; Fraunhofer diffraction theory; Retarded potential theory; Wave propagation in an inhomogeneous medium; Examples. 9. *Nonlinear waves*: Nonlinearity and quasilinear equations; Conservation equations; General effect of nonlinearity; Characteristics; Wavefronts bounding a constant state; Riemann invariants; Simple waves; The piston problem; Discontinuous solutions and shock waves; Examples. Answers. Index.

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